International Journal of Theoretical Physics, Vol. 20, No. 6, 1981

Inertial Frames of Reference: Mass Coupling to Space and Time

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Received September 20, 1980

We consider a new expression for the dependence of mass on velocity, more general than the corresponding law of the special theory of relativity (STR). The deviations from the STR become large with increasing rest mass. One should therefore measure the dependence of mass on velocity for objects with a large rest mass. The theory predicts that particles with real mass can travel with hyperlight velocities. The space-time picture discussed here is close to Mach's conception: It is assumed that the dynamical behavior of a particle in uniform translational motion is due to the "action" of all the other masses in the universe. Space-time is eliminated as an active cause and, in contrast to the STR, is not absolute within the theory discussed here. It turns out that effects based on the new transformation formulas (from the coordinates and time in a "stationary" frame to the coordinates and time in a "moving" frame) are identical to those expected from the Lorentz transformations. For example, it is known that rapidly "moving" μ mesons decay with a longer half-life than "stationary" μ mesons and the STR describes this effect quantitatively. However, there is no strong evidence for the validity of the STR because the theory given in this paper predicts the same result.

1. INTRODUCTION

Feinberg (1967) pointed out that the existence of particles (tachyons) which travel faster than light is not in contradiction to the special theory of relativity (STR) if their mass is an *imaginary* quantity. In this paper we discuss the possibility of faster-than-light particles with *real* mass. In this case the principles of the STR are violated. However, as remarked by Feinberg (1967), a particle which travels with hyperlight velocity does not involve logical inconsistencies provided that we are able to measure its position at two times and then calculate its velocity, by division, to be greater than the velocity of light.

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The STR is based on two postulates: (i) the principle of relativity and (ii) the principle of the constancy of the velocity of light. The laws derived from these two postulates are consistent and this holds also for the logical structure of the STR; in our opinion it is not possible to extend our knowledge of space and time without using more general assumptions than those given by (i) and (ii). If there exist faster-than-light particles with real mass it is necessary to construct a space-time theory which is more general than the STR.

In this paper we analyze the *uniform translational motion* of particles; we shall not touch upon problems associated with the general theory of relativity. The space-time theory given here is based on a new expression for the dependence of mass on velocity which is more general than the corresponding expression of the STR. From this expression follows that the deviations from the STR grow with increasing rest mass. One should therefore measure this law for objects (or systems) with a large rest mass.

Other relevant results are the following:

(1) The theory predicts that objects with real mass can travel with hyperlight velocities.

(2) The laws of the STR are included as a special case.

(3) Space-time forms a *nonabsolute* continuum within the theory discussed here. Within the STR space-time is an absolute one (Minkowski space). The statement "absolute" means (see Einstein, 1955, 1963) not only physically real, but also independent in its physical properties. As remarked by Einstein (1955, 1963), this is unsatisfactory because such a space-time only plays a determining role in all processes, without in its turn being influenced by them.

Till now no serious experiment shows that the STR is inadequate. However, it is shown in this paper that effects based on the transformation formulas (existing between the coordinates and time in a "stationary" frame S and the coordinates and time in another "moving" frame S') must be identical to those expected from the STR. For example, it is known that rapidly "moving" μ mesons decay with a longer half-life than "stationary" μ mesons, and the STR describes this effect quantitatively. However, in our opinion this is not strong evidence for the validity of the STR because the theory discussed in this paper predicts the same result.

2. DEPENDENCE OF MASS ON VELOCITY

Let us consider a particle moving at the velocity v relative to the "stationary" frame of reference with coordinates (x_1, x_2, x_3) and time t, and let us suppose that v is directed along the x_1 axis. The rate of change of momentum along the x_1 axis is given by the force F_{x_1} acting on the particle (Newton's second law): $F_{x_1}=dP/dt$. We define the mass m of the particle as the ratio of momentum P to its velocity:

$$m=P/v \tag{1}$$

Within the STR the mass varies with the velocity according to the law

$$m=\frac{m_0}{\left(1-v^2/c_0^{\,2}\right)^{1/2}} \tag{2}$$

In this section we derive an expression for the dependence of mass on velocity more general than that given by equation (2).

We define the *kinetic energy* E_k of the particle as follows: Its differential change is the force F_{x_1} times the differential distance moved:

$$dE_k=F_{x_1}dx_1 \tag{3}$$

The derivative of the kinetic energy with respect to the time is

$$\begin{aligned}\frac{dE_k}{dt}&=v\frac{dP}{dt}\\&=mv\frac{dv}{dt}+v^2\frac{dm}{dt}\end{aligned} \tag{4}$$

From this we obtain after some simple manipulations (provided $dm/dt\neq 0$)

$$\frac{dE_k}{dm}=\frac{m}{2}\frac{dv^2}{dm}+v^2 \tag{5}$$

Furthermore, we assume that the inverse function of $m=m(v)$ exists and that the derivative of the inverse function $v^2=\delta(m)$ is defined over the whole mass range of interest. Then we have $d\delta(m)/dm\cdot dm/dv^2=1$ and we may write equation (5) as follows:

$$\begin{aligned}\frac{dE_k}{dm}&=\frac{m}{2}\frac{d\delta(m)}{dm}+\delta(m)\\&=\frac{m}{2}\frac{1}{dm/dv^2}+v^2\end{aligned} \tag{6}$$

If we now integrate, we get

$$E_k = \int_{m_0}^{m} \left[\frac{m'}{2} \frac{d\delta(m')}{dm'} + \delta(m') \right] dm' \\ = \gamma(m) - \gamma(m_0) \tag{7}$$

where $m_0 = m(0)$ is the rest mass and

$$\gamma(m) = \int_{0}^{m} \left[\frac{m'}{2} \frac{d\delta(m')}{dm'} + \delta(m') \right] dm' \\ = E \tag{8}$$

$$\gamma(m_0) = \int_{0}^{m_0} \left[\frac{m'}{2} \frac{d\delta(m')}{dm} + \delta(m') \right] dm' \\ = E_0 \tag{9}$$

E is the total energy of the particle and E_0 its rest energy. The general expression for $\gamma(m)$ is not known. Let us continue by assuming that a Taylor expansion in powers of the mass m exists for $\gamma(m)$. The use of a Taylor expansion for the description of $\gamma(m)$ will be suitable for the discussion of effects which show deviations from the STR. Provided that the energy E of the particle vanishes if the "moving" mass m becomes zero (this does not mean that particles with a rest mass of zero ($m_0 = 0$) do not exist) we obtain for the Taylor expansion the expression

$$E = \sum_{k=1}^{\infty} m^k \gamma^{(k)} \frac{1}{k!} \tag{10}$$

where

$$\gamma^{(k)} = \left[\frac{d^k \gamma(m)}{dm^k} \right] \Bigg|_{m=0} \tag{11}$$

In particular, by means of equation (7) we get

$$\gamma^{(1)} = \left[\frac{m}{2} \frac{d\delta(m)}{dm} + \delta(m) \right] \Bigg|_{m=0}$$

$$\gamma^{(2)} = \frac{1}{2} \left[3 \frac{d\delta(m)}{dm} + m \frac{d^2\delta(m)}{dm^2} \right] \Bigg|_{m=0} \tag{12}$$

$$\cdots$$

$$\gamma^{(n)} = \frac{1}{2} \left[(n+1) \frac{d^{(n-1)}\delta(m)}{dm^{(n-1)}} + m \frac{d^n\delta(m)}{dm^n} \right] \Bigg|_{m=0}$$

If we set $\gamma^{(1)}=c_0^2$ in equation (10) and restrict ourselves to the first term in the expansion, we obtain the well-known result for the energy ($E=mc_0^2$) of the STR. If there exist terms with m^k, $k>1$, the results of the STR are only valid for sufficiently small masses.

Now let us approximate the energy E by the first two terms [see equation (10)]:

$$E=m\gamma^{(1)}+\tfrac{1}{2}m^2\gamma^{(2)} \tag{13}$$

The physical meanings of $\gamma^{(1)}$ and $\gamma^{(2)}$ are discussed below. The nonlinear term in equation (13) means, that, if the energy E is conserved, the mass m is not an additive quantity. Although the nonlinear term does not appear in the STR, also here the mass is *not* always an additive quantity: the mass of a system is not equal to the sum of the masses of the particles forming it if the velocities of the particles in the center-of-mass system are different (see Terletzkii, 1968).

With equation (13) we obtain for the differential equation (6)

$$\frac{dm}{dv^2}=\frac{m}{2}\,\frac{1}{\gamma^{(1)}+m\gamma^{(2)}-v^2} \tag{14}$$

The integration of equation (14) over all the masses from m_0 to m and over all the velocities from v_0 to v leads to the following result for the dependence of mass on velocity:

$$\begin{aligned} m&=m(v^2)\\ &=m_0\left(\frac{1+\frac{2}{3}\gamma^{(2)}/\gamma^{(1)}m_0-v_0^2/\gamma^{(1)}}{1+\frac{2}{3}\gamma^{(2)}/\gamma^{(1)}m-v^2/\gamma^{(1)}}\right)^{1/2} \end{aligned} \tag{15}$$

If we integrate from the mass m_0, at velocity $v=0$, to the mass m with the velocity v, we obtain

$$m=m_0\left(\frac{1+\frac{2}{3}\gamma^{(2)}/\gamma^{(1)}m_0}{1+\frac{2}{3}\gamma^{(2)}/\gamma^{(1)}m-v^2/\gamma^{(1)}}\right)^{1/2} \tag{16}$$

With $\gamma^{(1)}=c_0^2$ and sufficiently small masses ($m\ll\frac{3}{2}\gamma^{(1)}/\gamma^{(2)}$) we get the result of the STR [see equation (2)]. If we accept the STR as a special case of our theory $\gamma^{(1)}$ is given by c_0^2 and we obtain from equation (16)

$$\lim_{v\to c_0}\frac{m}{m_0}=\left(\frac{3}{2}\,\frac{c_0^2}{\gamma^{(2)}}\,\frac{1}{m_0}+1\right)^{1/3} \tag{17}$$

Within the STR m/m_0 is always infinite for $v=c_0$ and it is not possible to exceed c_0. Within the theory presented here *signals can propagate at a velocity higher than that of light* just in the case $\gamma^{(2)}>0$.

Velocity of Particles with a Rest Mass of Zero (e.g., Photons). By means of equation (16) we get for the velocity c_m of particles with a rest mass of zero the following expression:

$$c_m=\left(\gamma^{(1)}+\tfrac{2}{3}m\gamma^{(2)}\right)^{1/2} \tag{18}$$

or with equation (13)

$$c_m=\left\{\tfrac{1}{3}\gamma^{(1)}+\tfrac{2}{3}\left[2E\gamma^{(2)}+\left(\gamma^{(1)}\right)^2\right]^{1/2}\right\}^{1/2} \tag{19}$$

c_m has the property

$$\begin{aligned}\lim_{E\to 0} c_m{}^2 &= \gamma^{(1)} \\ &= c_0{}^2\end{aligned} \tag{20}$$

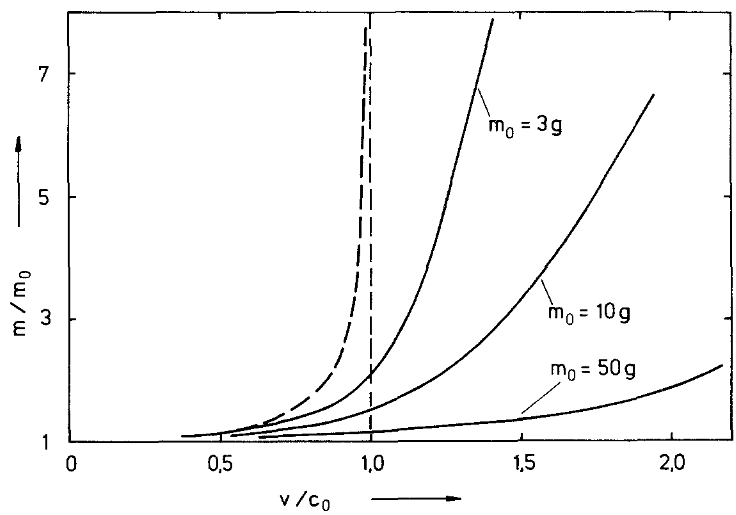

Fig. 1. Dependence of mass on velocity for several rest masses m_0 (full curves). The broken line shows the result of the STR. The constant $\gamma^{(2)}$ was chosen to be positive. The deviations from the STR become large with increasing rest mass. The effects become effective for $m_0>1$ g, and the corresponding energy is of the order of 10^{24} GeV. No accelerator can produce particles with energies like this. Therefore, the effects predicted here do not play a role in atomic physics and in the physics of elementary particles. However, the effects might be enormous in the case of astrophysical objects.

This means that c_m^2 approaches the fundamental constant $\gamma^{(1)}$ with decreasing energy (or mass). Within the STR the velocity of light is given by c_0 [see equation (2)].

Numerical Discussion. If we accept the STR as a special case of our approach $\gamma^{(1)}$ is given by $c_0^2=9\times10^{20}$ cm^2/sec^2. The rest mass of the neutron has been measured very precisely; the accuracy of $\pm0.0048\%$ is quoted in König et al. (1962). From this we have estimated an upper limit for the absolute value of the constant $\gamma^{(2)}$ by means of equation (13): $|\gamma^{(2)}|\leqslant5.7\times10^{19}$ erg/g^2. We have calculated the dependence of mass on velocity by means of equation (16) for several values of m_0. In principle, $\gamma^{(2)}$ can be positive or negative. The results are presented in Figures 1 and 2, respectively. Whereas within the STR the quantity m/m_0 only depends on the velocity v, the theory presented here predicts that m/m_0 depends on v *and* on the rest mass m_0. For example, in this case of $\gamma^{(2)}>0$ the curves become smooth with increasing m_0. In this case there is no upper limit for the velocity of objects with $m_0\neq0$.

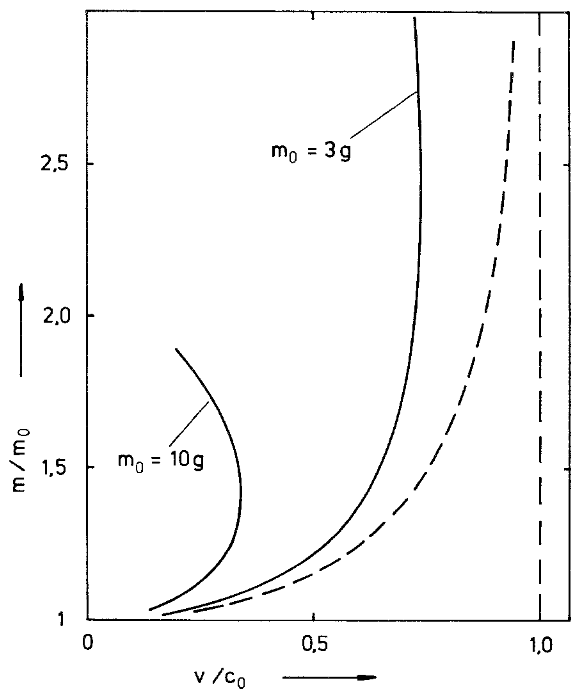

Fig. 2. Dependence of mass on velocity for several rest masses m_0 (full curves). The broken line shows the result of the special theory of relativity. The constant $\gamma^{(2)}$ was chosen to be negative.

3. CONSTRUCTION OF SPACE-TIME

We have derived the formula for the dependence of mass on velocity [equation (16)] *without* describing space-time. In this section we want to construct space-time on the basis of equation (16).

3.1. Transformation Formulas. We want to deduce the relations existing between the coordinates and time (x_1, x_2, x_3, t) in a "stationary" frame S and the coordinates and time (x_1', x_2', x_3', t') in another "moving" frame S'. S and S' are moving relative to each other with a uniform translational motion; v is the velocity of S' relative to S. As in Section 2 , v is directed along the x_1 axis. At this stage it is sufficient to know that the "stationary" frame S is described by clocks and scales. We shall give below a more detailed description of S.

Assumption 1. The properties of the transformation formulas are independent of the origin of the coordinates and of the origin of time. In this case the relations between (x_1, x_2, x_3, t) and (x_1', x_2', x_3', t') must be linear.

Assumption 2. The laws of electrodynamics are valid for *all* frames of reference.

The photon's velocity (its rest mass is zero) is $c_{m_{s0}}$. If in Maxwell's equations we replace the velocity of light c_0 by $(\gamma^{(1)} + \frac{2}{3} m_{s0} \gamma^{(2)})^{1/2}$ [see equation (18)] it is straightforward to show on the basis of Assumptions 1 and 2 that the transformations must have the following form:

$$
\begin{aligned}
x_1' &= \frac{x_1 - vt}{\left[1 - v^2/\left(\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s0}\right)\right]^{1/2}} \\
x_2' &= x_2 \\
x_3' &= x_3 \\
t' &= \frac{t - \left(\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s0}\right)^{-1} v x_1}{\left[1 - v^2/\left(\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s0}\right)\right]^{1/2}}
\end{aligned}
\tag{21}
$$

In the formulas (21) instead of $c_{m_{s0}}{}^2$ we have used the expression $(\gamma^{(1)} + \frac{2}{3} m_{s0} \gamma^{(2)})$.

The transformations (21) have the same structure as the Lorentz transformations, and, therefore, also in our theory the numerical value of the velocity of light (given in our theory by $c_{m_{s0}}$) remains unchanged when we proceed from S to S'. Thus Michelson's experiment would also be negative in the case discussed here. The meaning of the transformations (21), and in particular their dependence on m_{s0}, is discussed in detail below. From the formulas (21) we get the Lorentz transformations if $m_{s0} \ll \frac{3}{2}\gamma^{(1)}/\gamma^{(2)}$ and $c_0 = (\gamma^{(1)})^{1/2}$ [see equation (20)].

3.2. Determination of the Space-Time Metric in the "Stationary" Frame of Reference. In Section 3.1 we have deduced the relations existing between (x_1, x_2, x_3, t) and (x_1', x_2', x_3', t'). We have said nothing about the determination of the space-time metric in the "stationary" frame of reference S.

In order to determine the velocity $c_{m_{s0}} = (\gamma^{(1)} + \frac{2}{3}m_{s0}\gamma^{(2)})^{1/2}$ of the photon we have to measure in S its position at two times; Δx_1 is the difference of the positions and Δt is the difference of the times. From the transformation formulas (21) follows that $c_{m_{s0}}$ must be a *constant* (independent of the energy or the mass of the photon) whatever may be the motion of the source which emits the photon. Then the following relations have to be satisfied:

$$\begin{aligned} c_{m_{s0}} &= \frac{\Delta x_1}{\Delta t} \\ &= \text{const} \\ &= \left(\gamma^{(1)} + \tfrac{2}{3}m_{s0}\gamma^{(2)}\right)^{1/2} \end{aligned} \tag{22}$$

Equation (22) seems to involve a contradiction: On the one hand $c_{m_{s0}}$ must be constant and on the other hand $c_{m_{s0}}$ is dependent on m_{s0} and, therefore, on the energy $E = m_{s0}\gamma^{(1)} + \frac{1}{2}m_{s0}{}^2\gamma^{(2)}$. Because of the Doppler effect the energy and, therefore, the velocity of a photon emitted by a "moving" source must be different from that emitted by a "resting" source. This is in contradiction to $c_{m_{s0}} = \text{const}$ and, therefore, in contradiction to the transformation formulas (21). The *only* possibility of solving equation (22) is to assume that space-time forms a *nonabsolute* continuum. Within the STR space-time is definitely an absolute one (Minkowki's space). First we give a *formal* solution of equation (22) and secondly we shall discuss qualitatively a possible *physical conception*.

Formal Solution. The constancy of the velocity of light is required. The *only* possibility of fulfilling this law is to assume that the space-time metric in S is mass dependent: The distances and time intervals in the "stationary" frame of reference have to be constructed in such a way that equation (22) is satisfied. The following ways are possible: If we choose for all "stationary" frames of reference the *same* metric for the time (independent on m_{s0}), the distances Δx_1 must be dependent on the mass m_{s0}. With equation (22) we obtain

$$\Delta x_1 = \Delta x_1(m_{s0}) = \left(\gamma^{(1)} + \tfrac{2}{3} m_{s0} \gamma^{(2)}\right)^{1/2} \Delta t \tag{23}$$

Another possibility is to choose for all "stationary" frames of reference the *same* metric for the coordinates. In this case the time intervals Δt must be dependent on m_{s0} and we obtain

$$\Delta t = \Delta t(m_{s0}) = \left(\gamma^{(1)} + \tfrac{2}{3} m_{s0} \gamma^{(2)}\right)^{-1/2} \Delta x_1 \tag{24}$$

Either equation (23) or equation (24) can be used in order to fix the space-time geometry in a "stationary" frame of reference.

A given space-time metric in the "stationary" frame S is always coupled to a mass. In the following we shall call it the rest mass of the "stationary" frame S. Or the other way round: A given rest mass (called m_{s0}) produces its space-time metric; another rest mass $m_A \neq m_{s0}$ produces another space-time metric. From this point of view *space-time cannot be considered as an absolute continuum* and *we have to identify the "stationary" frame of reference with the reference mass*. The velocity of a photon with *any* energy (or mass) is given by $(\gamma^{(1)} + \frac{2}{3} m_{s0} \gamma^{(2)})^{1/2}$ within the space-time metric of m_{s0} [equations (23) and (24)].

The space-time metric determined by the rest mass m_{s0} are the *eigencoordinates* and the *eigentime* of m_{s0}. Examples are given in Figure 3. In order to fix the eigencoordinates and the eigentime for any mass in the laboratory we have to introduce a *standard* system (see the Appendix). The standard system can be fixed arbitrarily but should be the same in all laboratories. The formulas (21) transform the eigencoordinates x_1, x_2, x_3 and the eigentime t of the rest mass m_{s0} from S to S'. More details concerning the transformations (21) are given in Section 4; we shall see that the introduction of eigencoordinates and eigentime do not give rise to new measurable effects, and, therefore, the existence of a nonabsolute space-time should mainly be of interest for the theory of cognition.

Example. The distance of 1 cm measured in a "stationary" frame S_1 with the rest mass m_1 [in equation (23) m_{s0} is replaced by m_1] is longer than the distance of 1 cm in another "stationary" frame S_2 with the rest mass m_2 [in equation (23) m_{s0} is replaced by m_2] provided $m_1 > m_2$ and $\gamma^{(2)} > 0$. The velocity of a photon with the mass $m_A \neq m_1 \neq m_2$ (its energy is

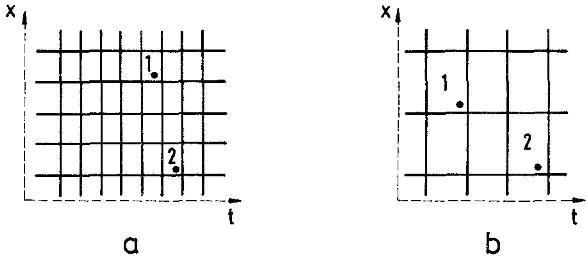

Fig. 3. Qualitative discussion of the space-time properties. Figure 1a and Figure 1b show the structure of the space-time for different masses m_a and m_b, respectively. Because $m_a \neq m_b$ the distances and time intervals are different from each other. There is a global validity of the metric in Figures 1a and 1b, respectively: in both cases the particles are not accelerated and we have for each space-time point (e.g., 1 and 2) the same particle mass ($m_{a1}=m_{a2}=m_a$, $m_{b1}=m_{b2}=m_b$) and, therefore, the same geometry.

$E=m_A\gamma^{(1)}+\frac{1}{2}m_A^2\gamma^{(2)})$ measured in S_1 is given by

$$c_{m_1}=\left(\gamma^{(1)}+\tfrac{2}{3}m_1\gamma^{(2)}\right)^{1/2} \tag{25}$$

and the velocity of the *same* photon measured in S_2 is

$$c_{m2}=\left(\gamma^{(1)}+\tfrac{2}{3}m_2\gamma^{(2)}\right)^{1/2} \tag{26}$$

Clearly, the velocity of the photon with the mass m_A measured in units of its eigencoordinates and eigentime [in equation (23) m_{s0} is replaced by m_A] is

$$c_{mA}=\left(\gamma^{(1)}+\tfrac{2}{3}m_A\gamma^{(2)}\right)^{1/2} \tag{27}$$

In other words, in this case the rest mass of the "stationary" frame is identically equal to the mass m_A of the photon.

Within the *fixed* space-time metric (for example, fixed by m_1) the velocity of light is constant and independent of its mass (or energy). This means that the velocity of a photon is independent of the velocity of the source which emits the photon [as required by the transformations (21)]. Clearly, because of Doppler's effect the energy (or mass) of a photon emitted by a "moving" source must be different from that emitted by a "resting" source. However, the velocity of the photon remains unchanged because this effect does not change the space-time metric in S_1.

Within the STR the metric (used in everyday life) can be fixed arbitrarily and is the same for all "stationary" frames of reference; the velocity of light is always $c_0=3\times10^{10}$ cm/sec. From our point of view the space-time metric used in everyday life is standardized by a mass of zero: note that the velocity of light c_0 used within the STR is related to $c_{m_{s0}}$ by [see equation (20)]

$$c_0 = \lim_{m_{s0} \to 0} c_{m_{s0}} \tag{28}$$

In conclusion, we can say that the velocity of light $c_{m_{s0}}$ [expressed by equation (22)] reflects a space-time property. It should be mentioned that within the STR the velocity of light $c_0 = (\gamma^{(1)})^{1/2}$ also reflects a space-time property: c_0 couples space and time. The additional term $\frac{2}{3} m_{s0} \gamma^{(2)}$, which appears in our theory, couples space-time to the mass m_{s0}.

Physical Concept. There is no observation that indicates that the coordinates and time in a "stationary" frame are mass dependent as required by the equations (23) and (24). This suggests we assume that space-time is *not* the *cause* of physically real effects (in contrast to Newton's and Minkowski's space). The following qualitative discussion is an example of how space-time can be eliminated as an active cause and play the role of an *auxiliary element* for the geometrical description of physically real properties.

In the previous discussion we have used the following physical elements: The mass m_{s0}, the coordinates x_1, x_2, x_3, and the time t. However, there are other masses m_i, $i = 1, 2, \ldots$, in the universe besides m_{s0}. We would like to assume that the dynamical behavior of any mass (for example, the mass m_{s0}) is due to the "*action*" of all the other masses. We say the particle interacts with all the others. What does "action" or "interaction" mean? It involves the knowledge of *what* makes the particle go (namely, all the other masses) without getting into the machinery of it. All that we can do is to describe *how* it moves (see also the discussion in Feyman et al., 1964). In other words, we are only able to form "*pictures*". We picture the interaction by means of a field φ, which is the *representation* of the interaction in space using the coordinates of the masses:

$$\varphi = \varphi(\mathbf{r}_{s0,1}, \mathbf{r}_{s0,2}, \ldots, \mathbf{r}_{s0,i}, \ldots) \tag{29}$$

where $\mathbf{r}_{s0,i}$ is the relative position vector from the mass m_{s0} to the particle i. The force $\mathbf{F}_{s0}$ acting on m_{s0} is proportional to the gradient of φ:

$$\mathbf{F}_{s0} \sim \operatorname{grad} \varphi \tag{30}$$

Since we are analyzing the unaccelerated motion we have

$$\mathbf{F}_{s0} = 0 \tag{31}$$

and

$$\mathbf{v} = \text{const} \tag{32}$$

From the right-hand side of equation (29) it follows that in this case the mass m_{s0} moves through a space-time region in which the field is *flat* (uncurved); within this space-time region φ is independent of the coordinates.

Note that within the frame of this conception the *cause* for the dynamical behavior is the mutual "action" of all the masses. Space-time appears as an *auxiliary element*; it represents geometrically the physically real properties.

The analysis of the field φ (in principle, it can involve the four kinds of fundamental interactions, in particular the gravitation) lies out of the scope of this paper and is not necessary for the understanding of the space-time description given in this paper.

In summary, we have analyzed the uniform translational motion of a mass m_{s0} as the interaction of m_{s0} with an uncurved field. We could not give the machinery of this interaction process. Clearly, in order to obtain a consistent picture this machinery must be compatible with the effects produced by the law for the dependence of mass on velocity [equation (16)]. It should be emphasized that other conceptions could interpret the role of a *nonabsolute* space-time.

Furthermore, we have discussed that the *cause* for the dynamical behavior of any particle is entirely the "action" of all the other masses. In the frame of this concept space-time must be considered as an *auxiliary element*; it geometrically describes physically real properties. That means space-time cannot give rise to any physically real effects and cannot be influenced by any physical conditions; in this sense the dependence of the space-time metric in S on m_{s0} [see equations (23) and (24)] reflects an *instruction* (how to fix the metric in the "stationary" frame S) and is not a measureable law. As we shall discuss below the elimination of space-time as an active cause is not in contradiction to the fact that the law for the slowing down of "moving" clocks gives rise to real effects. The meaning of the transformations (21) is discussed in Section 4.

Comparison to Space-Time of the STR. Within the STR space-time is absolute (the four-dimensional Minkowski space). Clocks and scales in a "stationary" frame S, which determine space and time, are independent of any mass and the metric is the same for all S. The statement "absolute" means (see the discussion in Einstein, 1955, 1963) that space-time is independent in his physical properties, having a physical effect, but not itself influenced by physical conditions, and this is unsatisfactory. Einstein

(1955, 1963) concludes:

It is contrary to the mode of thinking in science to conceive of a thing (space-time within the STR) which acts itself, but which cannot be acted upon. This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. According to him, a material particle does not move in unaccelerated motion relatively to space, but relatively to the centre of all the other masses in the universe; in this way the series of causes of mechanical phenomena was closed....

It is obvious that the physical conception outlined in this paper is very close to the ideas of Mach. Because of equation (22) we were able to give up the unsatisfactory conception of an absolute space-time; an absolute space-time would lead to contradictions within the theory discussed here.

3.3. Inverse Transformations. Both the "moving" and the "stationary" frames of reference are equivalent if they have the same metric. This is fulfilled if $m_{s0} = m_{s'0}$, where $m_{s'0}$ is the rest mass measured in S'. If S and S' are equivalent the inverse transformations from S' to S must be identically equal to the direct one [from S to S', see equation (21)]; they must only differ in the sign of the velocity:

$$\begin{aligned}
x_1 &= \frac{x_1' + vt'}{\left[1 - v^2/\left(\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s'0}\right)\right]^{1/2}} \\
x_2 &= x_2' \\
x_3 &= x_3' \\
t &= \frac{t' + \left(\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s'0}\right)^{-1} x_1'}{\left[1 - v^2/\left(\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s'0}\right)\right]^{1/2}},
\end{aligned} \tag{33}$$

where

$$m_{s'0} = m_{s0} \tag{34}$$

Since we have to identify the reference system with the reference body, the velocity v in equation (21) is determined by the mass m_s of the moving object and its rest mass m_{s0}. Rewriting equation (16) we obtain

$$v^2 = \left(\gamma^{(1)} + \tfrac{2}{3}\gamma^{(2)} m_s\right) - \left(\gamma^{(1)} + \tfrac{2}{3}\gamma^{(2)} m_{s0}\right)\left(m_{s0}^{\,2}/m_s^{\,2}\right) \tag{35}$$

Another way to determine the transformation formulas is the following: Terletskii (1968) discussed on the basis of very *general assumptions* (without the use of the postulate of the constancy of the velocity of light) the fact that the transformation formulas must have the form of equation (21). In his formulas, however, an unknown function appears instead of $(\gamma^{(1)} + \frac{2}{3}\gamma^{(2)}m_{s0})$, which has the dimensions of velocity squared. In the case discussed here Terletskii's unknown function should involve the constants $\gamma^{(1)}$ and $\gamma^{(2)}$, because there is no other way to introduce these constants in the formulas for the transformations of the coordinates and time. Terletskii's function can only be formed by the constants $\gamma^{(1)}$ and $\gamma^{(2)}$ in connection with a quantity which must have the dimensions of mass. In particular, the mass is coupled by the constant $\gamma^{(2)}$. From this point of view it seems to be natural to take ${c_{m_{s0}}}^2 = \gamma^{(1)} + \frac{2}{3}\gamma^{(2)}m_{s0}$, since no other quantity with the dimensions of the velocity squared enters the formula for the dependence of mass on velocity.

4. MEANING OF THE TRANSFORMATION FORMULAS

The transformation formulas (21) should have a precise meaning in analogy to the Lorentz transformations. In the theory given here different transformations exist for different masses.

Space is measured by scales and time is measured by clocks. We should be able to state, for example, what happens to a clock when it is carried around the world by plane; otherwise the theory is not related to the very precise tests of STR which are available today.

It is easy to show that the expression

$$\beta = v/c_{m_{s0}} \tag{36}$$

is independent of m_{s0}. Using equation (A.1) (see the Appendix) we obtain immediately

$$\begin{aligned}\beta &= \frac{v}{c_{m_{s0}}} = \frac{\Delta x_1}{\Delta t} \frac{1}{\left(\gamma^{(1)} + \frac{2}{3}m_{s0}\gamma^{(2)}\right)^{1/2}} \\ &= \frac{\Delta {x_1}^N}{\Delta t^N} \frac{1}{\left(\gamma^{(1)} + \frac{2}{3}m_N\gamma^{(2)}\right)^{1/2}}\end{aligned} \tag{37}$$

As pointed out in the Appendix the standard mass m_N is the same for all laboratories and can be fixed arbitrarily. If we choose $m_N = 0$ we obtain

$$\beta = v/c_0 \tag{38}$$

In this case v and c_0 can be measured by means of scales and clocks used within the STR (see Section 2).

The fact that β is independent of m_{s0} is important for the following reason: It is straightforward to show by means of the transformation formulas (21) that the law for the slowing down of clocks is given by

$$\tau/\tau_0 = (1-\beta^2)^{-1/2} \tag{39}$$

and the law for the contraction of moving scales by

$$l/l_0 = (1-\beta^2)^{1/2} \tag{40}$$

where τ_0 is the time interval measured by a "stationary" clock, τ is the interval measured by a "moving" clock, l_0 is the length of a "stationary" scale, and l is the length of a "moving" scale. Since β is independent of m_{s0} we may use for it the expression given by equation (38). This means that the laws for the slowing down of clocks and the contraction of moving scales predicted here are *identical* to the corresponding results of the STR.

Clearly, in our theory τ_0 and l_0 are dependent on m_{s0} and this is due to the fact that space-time cannot be an absolute continuum within the theory given here. However, as already mentioned above, this kind of mass dependence reflects an *instruction* and is not a directly measurable law. This is because we have eliminated space-time as an active cause.

If we take a clock and a scale used in everyday life (defined in section 3.2) and jump on a train we observe for τ and l exactly the same results as those expected from the STR. For example, it is known that rapidly "moving" μ mesons decay with a longer half-life than "stationary" μ mesons and the STR describes this effect quantitatively. However, this is not strong evidence for the validity of the STR because the theory discussed in this paper predicts the same result.

The reason why we observe the same effects (concerning clocks and scales) in both theories is this: The mass dependence of the transformation formulas is due to the mass dependence of the space-time metric in S; the transformation of a *fixed* space-time (x_1, x_2, x_3, t) from S to S' (x_1', x_2', x_3', t') is independent of m_{s0} and is given by the Lorentz transformations. This fact means that beside τ/τ_0 and l/l_0 the transformation of the properties (for example, the electromagnetic field) of all other physical processes can only be dependent on β and not on the mass m_{s0}. Although the space-time discussed in this paper is fundamentally different from the space-time of the STR we obtain the following important result: *effects based on the transformation formulas* (21) *are identical to those expected from the STR*; *the mass dependence of the space-time metric in S only reflects an instruction*.

Although we have eliminated space-time as an active cause we observe that the slowing down of "moving" clocks is a physically real effect. This fact does not reflect a contradiction for the following reason: space-time describes physically real laws (in S and S'), and the machinery which makes the clocks go are also controlled by these real laws.

The space-time conception given here is based on a new expression for the dependence of mass on velocity [equation (16)]. In order to show up experimentally the differences between equation (16) and the corresponding law of the STR [equation (2)] one should measure the mass as a function of velocity. On the one hand the STR should be a good approximation in the case of [see equation (18)]

$$m \ll \frac{3}{2} \frac{\gamma^{(1)}}{\gamma^{(2)}} \tag{41}$$

On the other hand in order to obtain large differences between the laws for the dependence of mass on velocity [equations (2) and (16)] one should investigate objects with *large* masses ($m_0 > 1$ g, see Figure 1). Thus, we believe that astrophysical observations should be important for the detection of faster-than-light particles (or systems) with real mass.

However, it is straightforward to show (using Maxwell's equations) that *charged* objects cannot be accelerated through the "light barrier" ($v = c_{m_{s0}}$). This is because a "moving" charged particle (in the "stationary" frame described by an electrical field) produces a magnetic field and, because of the transformations (21), the energy of this field becomes infinite at $v = c_{m_{s0}}$. In conclusion, a *charged* object can exist at $v < c_{m_{s0}}$ and $v > c_{m_{s0}}$ but not at $v = c_{m_{s0}}$.

As far as the author knows, all the experimental proofs for the law that no particle can exceed the velocity of light are based on investigations of "moving" *charged* particles with relatively small masses. However, the theory given in this paper also predicts that charged particles cannot be accelerated through the "light barrier"; in this case on the effects due to equation (16) is superimposed the production of a magnetic field whose energy becomes infinity at $v = c_{m_{s0}}$.

5. SUMMARY AND FINAL REMARKS

(1) We have derived a new expression for the dependence of mass on velocity [equation (16)] without using any space-time conception. This expression is more general than the corresponding law of the STR [equation (2)]. On the one hand the deviations from the STR are getting large with

increasing mass m. On the other hand in the case of $m \ll \frac{3}{2}\gamma^{(1)}/\gamma^{(2)}$ the law of the STR [equation (2)] is a good approximation. Thus, one should measure the dependence of mass on velocity for objects (or systems) with a large mass; the effects might be enormous in the case of astrophysical objects. The effects will not play a role in atomic physics and the physics of elementary particles (see Figure 1).

It is easy to show that faster-than-light objects are possible if the function $\Delta(m) = E - mc_0^2$ (in this paper approximated by $\frac{1}{2}m^2\gamma^{(2)}$) has the property $d\Delta(m)/dm > 0$. However, charged objects cannot be accelerated through the "light barrier" ($v = c_{m_{s0}}$) but can exist either at $v < c_{m_{s0}}$ or at $v > c_{m_{s0}}$. There is a certain similarity with the phenomenon of resonance: in this sense the velocity of light can be considered as "resonant velocity" and is not an upper limit for the propagation of signals.

(2) Einstein's original derivation of $E = mc_0^2$ was from the Lorentz transformations, and the Lorentz transformations were derived from Maxwell's equations. In this paper we have chosen the following way: First we have derived the expression $E = m\gamma^{(1)} + \frac{1}{2}m^2\gamma^{(2)}$ [equation (13)]. Secondly we have *modified* Maxwell's equation as follows: We have replaced the velocity of light c_0 by $(\gamma^{(1)} + \frac{2}{3}m_{s0}\gamma^{(2)})^{1/2}$. Thirdly we have derived the transformations (21) from the modified Maxwell equations. In our approach we have to identify the reference system with the reference body and the velocity v in the transformations (21) is determined by the mass m_s of the moving object and its rest mass m_{s0} [see equation (35)]. Therefore, it is not possible to deduce the law for the dependence of mass on velocity from the transformations because the transformations can only be formulated by means of this law.

(3) Within the space-time picture discussed here accelerations cannot be described because space-time is uncurved. This is due to the fact that space-time is coupled to the mass; the mass of an accelerated object varies with time and therefore also the space-time metric varies with time, leading to a curved space-time.

The properties of each particle are given by (x_1, x_2, x_3, t, m). In other words, we have a five-dimensional representation. How can we determine the space-time metric of a frame in the case of two particles (called 1 and 2) with different masses? This question can only be answered on the basis of a five-dimensional algebra. Particle 1 is characterized by $(x_{11}, x_{21}, x_{31}, t_1, m_1)$ and particle 2 by $(x_{12}, x_{22}, x_{32}, t_2, m_2)$. From this we can define the properties of the frame for both particles by $(x_{11}, x_{21}, x_{31}, t_1, m_1) + (x_{12}, x_{22}, x_{32}, t_2, m_2) = (x_{13}, x_{23}, x_{33}, t_3, m_3)$. This can easily be done on the basis of the conservation laws. Within our approach accelerations (which is equivalent to a curved space-time) are not possible because space-time is flat. A five-dimensional algebra is needed on the basis of a *curved* space-time.

(4) It turns out in this theory that space-time cannot form an absolute continuum. We obtain a faithful picture if we assume that space-time is eliminated as an active cause in physics; how space-time could possibly be eliminated is discussed, a conception very similar to Mach's ideas: It is assumed that the dynamical behavior is due to the mutual "action" of all the masses in the universe. Space-time is the *geometrical representation* of the physically real laws and plays the role of an *auxiliary element*. It makes no sense to ask "How large is the universe?" or questions like this. We can only say something about *distances* (between the masses) and *time intervals*.

In contrast to the theory discussed here, space-time forms an absolute continuum within the STR; this kind of space-time has often been criticized (see, for example, Einstein, 1955, 1963) because it plays a determining role in all processes, without in its turn being influenced by them.

It has been demonstrated by Gödel (1949) that also within the *general* theory of relativity (GTR) the absoluteness of space-time is not eliminated. Gödel showed that within the GTR *absolute* rotations are possible: The *whole* universe (all the masses) can rotate within an absolute space-time.

(5) Although the space-time discussed in this paper is fundamentally different from the space-time of the STR we have found that effects based on the transformation formulas (21) are identical to those expected from the STR (for example, the behavior of the half-life of "moving" μ mesons).

(6) If the velocity squared exceeds the value $(\gamma^{(1)} + \frac{2}{3} m_{s0} \gamma^{(2)})$ in the transformation formulas (21), the coordinates and time become imaginary. It may be objected that imaginary coordinates and an imaginary time have no sense. However, in our theory space-time is not the cause for physical effects and, therefore, we have to investigate whether the physically real properties expressed by imaginary coordinates and an imaginary time make sense. In a first step we were able to show that causal anomalies cannot be used as an argument against the existence of faster-than-light particles with real mass. It should be mentioned that Feinberg (1967) already pointed out that the introduction of tachyons (faster-than-light particles with imaginary mass) does not lead to logical and causal anomalies. In the case of tachyons too the coordinates and time become imaginary in the "moving" frame of reference.

APPENDIX

In contrast to the STR, space-time is not absolute within the theory presented here. The coordinates and times for "stationary" systems with different masses are not equivalent. In order to fix these *eigencoordinates* and *eigentimes* in any "stationary" system we have to introduce a "stationary" standard system S^N consisting of the standard mass m_N and an arbitrarily fixed eigencoordinate–eigentime set of numbers x_1^N, x_2^N, x_3^N, t^N. From this it is easy to fix the space-time in any other "stationary" system S with the mass m_{s0}. Using both the distances and time intervals of the standard system and of any other system S, we obtain with equation (22) the following relation:

$$\frac{\Delta x_i}{\Delta x_i^N} = \left(\frac{\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s0}}{\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_N} \right)^{1/2} \frac{\Delta t}{\Delta t^N}, \qquad i=1,2,3 \tag{A.1}$$

There are two possibilities to fix the space-time in the system S:

(i) The *eigentimes* of all "stationary" systems are equivalent, i.e., the time does not depend on the mass m_{s0}. Then we obtain from equation (A.1)

$$\Delta x_i = \left(\frac{\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s0}}{\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_N} \right)^{1/2} \Delta x_i^N, \qquad i=1,2,3$$

$$\Delta t = \Delta t^N \tag{A.2}$$

For example, in the case of $\gamma^{(2)} > 0$ we obtain for

$m_{s0} > m_N$: *contraction* of space in system S relative to the space in the standard system.

$m_{s0} < m_N$: *extension* of space in system S relative to the space in the standard system.

(ii) The *eigencoordinates* of all "stationary" systems are equivalent, i.e., the space does not depend on the mass m_{s0}. In this case it follows from Equation (A.1)

$$\Delta x_i = \Delta x_i^N, \qquad i=1,2,3$$

$$\Delta t = \left(\frac{\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_N}{\gamma^{(1)} + \frac{2}{3}\gamma^{(2)} m_{s0}} \right)^{1/2} \Delta t^N \tag{A.3}$$

For example, in the case of $\gamma^{(2)} > 0$ we obtain for

$m_{s0} > m_N$ *slowing down* of clocks in system S relative to the clocks in the standard system.

$m_{s0} < m_N$: *slowing up* of clocks in system S relative to the clocks in the standard system.

To fix space-time in any "stationary" system with the mass m_{s0} we may use either method (i) or method (ii). Within the STR the space-time metric is the same for all "stationary" frames of reference.

REFERENCES

Einstein, A. (1955). *The Meaning of Relativity*. Princeton University Press, Princeton, New Jersey.

Einstein, A. (1963). *Relativitätstheorie*, Vieweg Verlag, Braunschweig.

Feinberg, G. (1967). *Physical Review*, **159**, 1089.

Feynman, R. P., Leighton, R. B., and Sands, M. (1964). *The Feynman Lectures on Physics*. Addison-Wesley, Reading, Massachusetts.

Gödel, K. (1949). *Reviews of Modern Physics*, **21**, 447.

König, L. A., Mattauch, J. H. E., and Wopstra, A. H. (1962). *Nuclear Physics*, **31**, 18.

Terletzkii, Y. P. (1968). *Paradoxes in the Theory of Relativity*. Plenum Press, New York.